486: PART A

1) $f(x) = 7 - 15x + 9x^2 - x^3$

a) $7 - 15x + 9x^2 - x^3 = 0$

1	-1	9	-15	7
		-1	8	-7
	-1	8	-7	0

$(x-1)(-x^2 + 8x - 7) = 0$
$(x-1)(x^2 - 8x + 7) = 0$
$(x-1)(x-1)(x-7) = 0$
$x = 1, x = 7$

b) $f(2) = 7 - 15(2) + 9(4) - 8 = 5$

$f'(x) = -15 + 18x - 3x^2$
$f'(2) = -15 + 18(2) - 3(4) = 9$

Equation of tangent:
$y - 5 = 9(x-2)$
$y - 5 = 9x - 18$
$y = 9x - 13$

c) $f''(x) = 18 - 6x$
$f''(x) = 0$
$18 - 6x = 0$
$6x = 18$
$x = 3$

$f''(x)$: + | 3 | −

$x = 3$

2) $f(x) = \dfrac{9x^2 - 36}{x^2 - 9}$

a) symmetric about the y axis since $f(-x) = f(x)$

b) Vertical asymptote: $x^2 - 9 = 0$
$(x-3)(x+3) = 0$
$x = -3, x = 3$

$\lim_{x \to 3^+} f(x) = \infty$ $\quad \lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = -\infty$ $\quad \lim_{x \to -3^-} f(x) = \infty$

Horizontal asymptote: $\lim_{x \to +\infty} \dfrac{9x^2 - 36}{x^2 - 9} = \lim_{x \to +\infty} \dfrac{9 - 36/x^2}{1 - 9/x^2} = \dfrac{9-0}{1-0} = 9$

$\lim_{x \to -\infty} \dfrac{9x^2 - 36}{x^2 - 9} = 9$

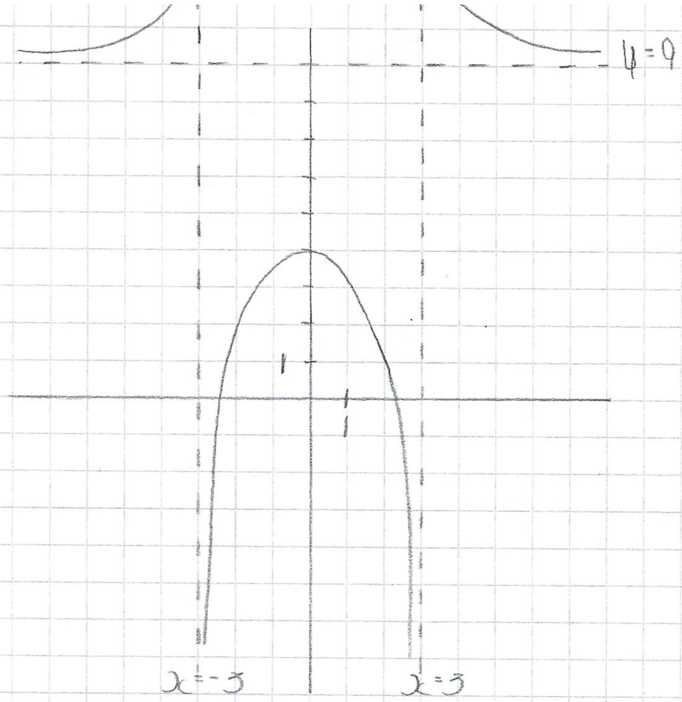
$y = 9$

c) $f'(x) = \dfrac{(x^2-9)(18x) - (9x^2-36)(2x)}{(x^2-9)^2} = \dfrac{18x^3 - 162x - 18x^3 - 72x}{(x^2-9)^2} = \dfrac{-234x}{(x^2-9)}$

$f'(x) = 0$
$\dfrac{-234x}{(x^2-9)^2} = 0$
$x = 0, \; x^2 - 9 \neq 0$
$x^2 = 9 \Rightarrow x = \pm 3$

$f'(x)$: + | −3 | + | 0 | − | 3 | −

increasing on $(-\infty, 0)$



3) $a(t) = \frac{1}{t}$, $v(1) = -2$, $x(1) = 4$, $t \geq 1$

a) $v(t) = \int a(t)\,dt$
$v(t) = \int \frac{1}{t}\,dt$
$v(t) = \ln t + c$
$v(1) = -2$
$-2 = \ln 1 + c$
$c = -2$

$v(t) = \ln t - 2$

b) $x(t) = \int v(t)\,dt$
$x(t) = \int (\ln t - 2)\,dt$
$x(t) = t\ln t - t - 2t + c$
$x(1) = 4$
$4 = \ln 1 - 3 + c$
$c = 7$

$x(t) = t\ln t - 3t + 7$

$\int \ln t\,dt$
let $u = \ln t$ $\quad \frac{dv}{dt} = 1$
$\frac{du}{dt} = \frac{1}{t}$ $\quad v = t$

$\ln t(t) - \int t\left(\frac{1}{t}\right)dt = t\ln t - \int dt = t\ln t - t + c$

c) $v(t) = 0$
$\ln t - 2 = 0$
$\ln t = 2$
$t = e^2$

$v(t)$: $\underset{1}{\cdot}$ $-$ $\underset{e^2}{\cdot}$ $+$
$\searrow$ _ $\nearrow$

$x(e^2) = e^2 \ln e^2 - 3e^2 + 7 = 2e^2 - 3e^2 + 7 = 7 - e^2 = -0.389$

$x(1) = -3 + 7 = 4$

$t = e^2$

36: PART B

i) $f(x) = \begin{cases} |x-1|+2, & \text{for } x<1 \\ ax^2+bx, & \text{for } x \geq 1 \end{cases}$

a) $f(x) = \begin{cases} |x-1|+2, & \text{for } x<1 \\ 2x^2+3x, & \text{for } x \geq 1 \end{cases}$

$x \geq 1,\ f(x) = x-1$
$x < 1,\ f(x) = -(x-1) = -x+1$

$\lim_{x \to 1^-} (-x+1)+2 = \lim_{x \to 1^-} (-x+3) = -1+3 = 2$

$\lim_{x \to 1^+} (2x^2+3x) = 2+3 = 5$

NO, $f(x)$ is not continuous since $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$

b) If continuous, $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\lim_{x \to 1^+} (ax^2+bx) = \lim_{x \to 1^-} (-x+1+2)$

$a+b = -1+3$

$a+b = 2$

c) If differentiable, $\lim_{x \to 1^-} \left[\frac{-x+1+2-(a+b)}{x-1}\right] = \lim_{x \to 1^+} \left[\frac{ax^2+bx-(a+b)}{x-1}\right]$

$= \lim_{x \to 1^-} \left[\frac{-x+3-(a+b)}{x-1}\right] = \lim_{x \to 1^+} \left[\frac{ax^2+(2-a)x-2}{x-1}\right]$

$= \lim_{x \to 1^-} \left[\frac{-x+3-2}{x-1}\right] = \lim_{x \to 1^+} \left[\frac{ax^2-ax+2x-2}{x-1}\right]$

$= \lim_{x \to 1^-} \left[\frac{-(x-1)}{(x-1)}\right] = \lim_{x \to 1^+} \left[\frac{ax(x-1)+2(x-1)}{x-1}\right]$

$= -1$

$= \lim_{x \to 1^+} \left[\frac{(ax+2)(x-1)}{x-1}\right]$

$= \lim_{x \to 1^+} (ax+2)$

$= a+2$

$\therefore a+2 = -1$

$a = -3 \Rightarrow b = 5$

OR: $\frac{d}{dx}(1-x+2) = \frac{d}{dx}(ax^2+bx)$

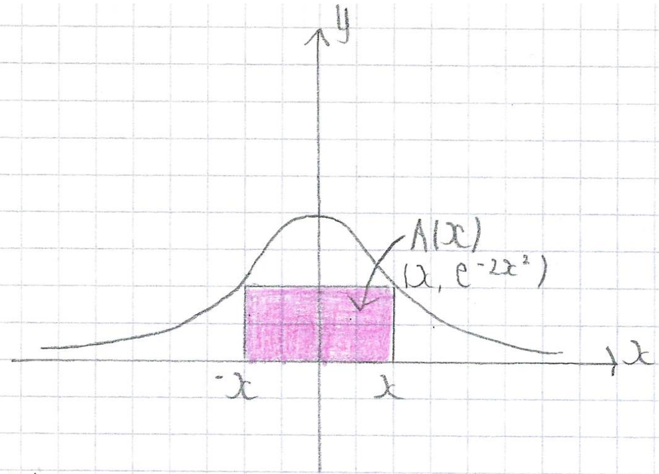
$-1 = 2ax+b$

$x=1$: $2a+b = -1$

$a+b = 2$

$\therefore a = -3,\ b = 5$

5)



a) $A(x) = 2x e^{-2x^2}$
$A(1) = 2e^{-2}$

b) $A'(x) = 2e^{-2x^2} + 2x(-4x)e^{-2x^2}$
$= 2e^{-2x^2} - 8x^2 e^{-2x^2}$

$A'(x) = 0$
$2e^{-2x^2} - 8x^2 e^{-2x^2} = 0$
$e^{-2x^2} - 4x^2 e^{-2x^2} = 0$
$e^{-2x^2}(1 - 4x^2) = 0$
$e^{-2x^2} \neq 0, \; 1 - 4x^2 = 0$
$4x^2 = 1$
$x^2 = \frac{1}{4}$
$x = \pm\frac{1}{2}$

$A'(x)$: $-$ at $-\frac{1}{2}$, $+$, at $\frac{1}{2}$, $-$
\ _ / \

Max when $x = \frac{1}{2}$

Greatest value $= 2\left(\frac{1}{2}\right)e^{-2(1/4)} = e^{-1/2} = \frac{1}{\sqrt{e}}$

c) $A(x)_{Av} = \frac{1}{2-0}\int_0^2 A(x)\,dx$
$= \frac{1}{2}\int_0^2 2x e^{-2x^2}\,dx$
$= \frac{1}{2}\left[-\frac{1}{2}e^{-2x^2}\right]_0^2$
$= \frac{1}{2}\left[-\frac{1}{2}e^{-8} + \frac{1}{2}e^0\right]$
$= -\frac{1}{4}e^{-8} + \frac{1}{2}$
$= -\frac{1}{4e^8} + \frac{1}{2}$

6)
a) Intersection: $\tan^2 x = \frac{1}{2}\sec^2 x$
$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{2}\left(\frac{1}{\cos^2 x}\right)$
$2\sin^2 x\cos^2 x = \cos^2 x$ → can divide by $\cos^2 x \neq 0$
$2\sin^2 x = 1$
$\sin^2 x = 1/2$
$\sin x = \pm 1/\sqrt{2}$
$x = \pi/4$

Area $R = \int_0^{\pi/4}$